Measuring Foreign Exposure*

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Abstract

We introduce a model-based measure of exposure to foreign shocks that incorporates propagation via high-order trade. The model implies that the response of output to foreign shocks (i.e., foreign exposure) can be approximated by a simple decomposition of output according to the location of final demand. We perform the decomposition using sector data on international input-output trade and evaluate its empirical merits relative to common alternatives often used to capture foreign exposure.

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1 Introduction

It has proved difficult to measure the exposure of a country or a sector to foreign shocks. At country level, a common approach consists in computing the ratio of *direct* exports (and sometimes imports) to GDP, a measure that is intuitive but has little support in theory especially in a world of global value chains. At sector level, a common approach consists in approximating implicit trade costs on the basis of observed trade flows, but low trade costs do not necessarily mean high exposure to foreign shocks. And while an important literature has recently emerged to characterize and summarize the features of *indirect* trade, its main purpose has not been to measure exposure to foreign shocks via vertical linkages.

In this paper we propose a model-based measure of foreign exposure accounting for propagation via vertical trade and easy to implement empirically at country or at sector level. We show in a multi-sector multi-country canonical model that the response of value added to foreign shocks can be approximated by the value of domestically produced goods sold to final consumers abroad, directly *or indirectly*. We compute this ratio for a large cross-section of country-sectors and document its properties and correlates, especially as compared with common alternatives. An attractive feature of our measure, which we label "HOT" for "high order trade", is that it extends naturally to sectors that are customarily assumed to be immune to foreign shocks for lack of much direct trade, e.g., services.

We start with a canonical model of international vertical trade with rich sector linkages. The model's equilibrium implies that the response of value added to foreign shocks is approximated by the share of output sold to foreign demand, what we call "HOT". We confirm the quality of this approximation through simulations that establish a close correlation between HOT and the response of value added to shocks. We also compare the performance of HOT in that respect with that of other common measures of "openness". In our simulations HOT always correlates positively with the simulated response of value added to foreign shocks, but common alternatives do not. This is not necessarily surprising because these alternative measures were not designed to capture foreign exposure; But in practice they are often used as such and the simulation results provide a warning against doing so.

The model implies that HOT approximates the response of value added to foreign shocks. We next test this directly in the data. Firstly we estimate the empirical correlation between HOT and country-sector fluctuations in value added. Secondly we estimate the empirical correlation between HOT and the international synchronization of economic fluctuations at sector level. And thirdly we estimate the empirical correlation between HOT and country-sector growth (reasoning that accumulated foreign supply shocks ought to result in long run growth). HOT is systematically positive and significant, including for services. As in the simulation exercise we also include conventional measures of "openness" in these three estimations: The coefficients are mostly not significantly different from zero. We conclude that HOT provides a sector-level measure of foreign exposure that is grounded in theory and uniquely validated empirically.

HOT is also easy to calculate from readily available input-output data. It is derived from the identity at the heart of input-output tables equating gross output to the sum of its final uses. We decompose output into its final uses that are purely domestic and those that are not. This involves two infinite sums: One that isolates output's purely domestic uses and one that contains all the others. The former sum summarizes all the ways in which the sector's output reaches final demand staying strictly within the same country. The latter includes all the ways borders are crossed down the supply chain, at any order: It embeds offshore outsourcing, in which segments of the supply chain are localized in different countries.¹

HOT computes the share in total output of the latter infinite sum and takes high values close to one if most of the sector's final uses are located across the border. In practice, it is the infinite sum focused on domestic sales that we identify in the data, by manipulating the world inputoutput matrix to isolate the purely domestic components of vertical trade. This manipulation was first introduced by Miller (1966) to compute an interregional feedback index at country level. It also follows the "hypothetical extraction" approach pioneered by Los, Timmer, and De Vries (2016), who perform similar decompositions to isolate various components of trade. Hummels, Ishii, and Yi (2001), Bems, Johnson, and Yi (2010) use a similar measure to dissect the great trade collapse of 2008-2009. Bems and Johnson (2017) use it to introduce value added exchange rates. Tintelnot et al. (2021) introduce a measure similar to ours in Belgian firm-to-firm data to study how international trade affects wages and unit costs at firm level.² To our knowledge, no one has shown, theoretically or empirically, that this decomposition can approximate foreign exposure.

A large literature uses Leontief inverses to account for indirect trade. Most prominently, "Trade in Value Added" (TiVA) exploits input-output linkages to obtain a measure of trade that is commensurate with national accounts, i.e., expressed in terms of value created rather than gross output (see for instance Johnson and Noguera, 2012, Koopman et al., 2014, Bems and Kikkawa, 2021, or Bems and Johnson, 2017). Our objective is different: While this literature introduces a measure of trade that is consistent with national accounts, we introduce a measure of foreign exposure that is consistent with theoretical propagation channels. Recently, Baldwin, Freeman, and Theodorakopoulos (2022) review the range of possible measures of exposure that can be obtained from data on global supply chains, including the one proposed in this paper.³

¹Input-output tables are silent about firm boundaries, so that HOT can in fact correlate with the existence of multinational companies. See Fally and Hillberry (2018), Alfaro et al. (2019), or Atalay et al. (2019).

²In the terminology set out by Antràs and Chor (2021), our approach is "macro" by nature since we examine measures of foreign exposure across countries and sectors. The complementary "micro" approach based on firm-level information still presents some limitations, since "there remains significant hurdles to linking micro datasets across countries" for instance because of confidentiality or compatibility issues (Antràs and Chor, 2021, Section 2.2).

³See also Baldwin and Freeman (2022) and Baldwin, Freeman, and Theodorakopoulos (2023).

In spite of recent advances, and as far as we can measure it, direct international trade in services remains small. The majority of service trade is domestic, which can be taken to suggest that services are by and large insulated from foreign developments. But of course foreign exposure depends on high order trade linkages: A firm selling services to a domestic export champion is highly, if indirectly, exposed to foreign shocks. There is no reason to expect that services (or more generally so called non-traded sectors) should have systematically lower foreign exposure than manufacturing sectors. For example, Johnson (2014) shows service trade is larger in value-added terms than in gross terms, which reflects the fact that service trade is mostly indirect across borders. So direct international trade is essentially silent about foreign exposure, especially as regards services. Measuring foreign exposure using HOT is particularly adequate for services: HOT introduces a precise measure, grounded in theory, readily available from input-output data, and that takes into account the indirect linkages of propagation that are especially relevant for services.

2 The model

This Section presents a multi-country, multi-sector model with input-output linkages adapted from Huo et al. (2021) and amenable to simulation. We first present the building blocks of the model. We then introduce foreign shocks and simulate the responses of output, of HOT, and of alternative indicators often used to measure foreign exposure at sector level. We examine the correlations between the simulated responses of output and the simulated responses of these indicators.

2.1 Building blocks

Production in sector r of country i is given by

$$\mathbf{Y}_i^r = \mathbf{Z}_i^r \left[(\mathbf{H}_i^r)^{\alpha^r} (\mathbf{K}_i^r)^{1-\alpha^r} \right]^{\eta^r} (\mathbf{M}_i^r)^{1-\eta^r},$$

where Z_i^r is a supply shock, H_i^r denotes labor input, K_i^r is capital input, and the intermediate input $M_i^r \equiv \left(\sum_j \sum_s (\mu_{ji}^{sr})^{\frac{1}{\epsilon}} (M_{ji}^{sr})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$; μ_{ji}^{sr} is an exogenous shifter and ϵ is the elasticity of substitution between varieties of the intermediate goods. Throughout the paper, subscripts denote countries and superscripts denote sectors. Both indexes are ordered so that the first identifies the location of production, and the second identifies the location of use. Capital is predetermined.⁴ Cost minimization implies

$$W_i^r H_i^r = \alpha^r \eta^r P_i^r Y_i^r,$$

$$P_{ji}^{sr} M_{ji}^{sr} = \xi_{ji}^{sr} (1 - \eta^r) P_i^r Y_i^r,$$

where W_i^r denotes the wage in (i, r), P_{ji}^{sr} is the price of the intermediate input produced in sector s of country j and used in sector r of country i, and P_i^r is the price of output in sector r of country i. The expenditure share ξ_{ji}^{sr} is given by

$$\xi_{ji}^{sr} = \frac{\mu_{ji}^{sr} (\tau_{ji}^s \mathbf{P}_j^s)^{1-\epsilon}}{\sum_{k,l} \mu_{ki}^{lr} (\tau_{ki}^l \mathbf{P}_k^l)^{1-\epsilon}},$$

where τ_{ji}^{s} denotes transport cost for sector *s* between countries *j* and *i*. Cost minimization implies that $\xi_{ji}^{sr} = \frac{P_{ji}^{sr} M_{ji}^{sr}}{P_{i}^{r} M_{i}^{r}}$. Transport costs are such that $P_{ji}^{sr} = P_{ji}^{s} = \tau_{ji}^{s} P_{j}^{s}$.

Households choose consumption to maximize $U\left(C_i - \sum_r (H_i^r)^{1+\frac{1}{\psi}}\right)$ subject to $P_i^c C_i = \sum_r W_i^r H_i^r + \sum_r R_i^r K_i^r$, where

$$C_{i} = \left[\sum_{j} \sum_{s} (\nu_{ji}^{s})^{\frac{1}{\rho}} (C_{ji}^{s})^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}},$$
$$P_{i}^{c} = \left[\sum_{j} \sum_{s} (\nu_{ji}^{s}) (P_{ji}^{s})^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

 P_i^c is the consumption price index, ρ is the elasticity of substitution between final goods, and R_i^r denotes the rental rate of capital. Optimal labor supply is given by

$$\mathbf{H}_{i}^{r} = \frac{\psi}{1+\psi} \left(\frac{\mathbf{W}_{i}^{r}}{\mathbf{P}_{i}^{c}}\right)^{\psi}.$$

Optimal expenditure shares in the final good are given by

$$\pi_{ji}^{s} = \frac{\nu_{ji}^{s} (\tau_{ji}^{s} \mathbf{P}_{j}^{s})^{1-\rho}}{\sum_{k,l} \nu_{ki}^{l} (\tau_{ki}^{l} \mathbf{P}_{k}^{l})^{1-\rho}} = \frac{\mathbf{P}_{ji}^{s} \mathbf{C}_{ji}^{s}}{\sum_{k,l} \mathbf{P}_{ki}^{l} \mathbf{C}_{ki}^{l}} = \frac{\mathbf{P}_{ji}^{s} \mathbf{C}_{ji}^{s}}{\mathbf{P}_{i}^{c} \mathbf{C}_{i}}.$$

Market clearing imposes that

$$\mathbf{P}_i^r \mathbf{Y}_i^r = \sum_s \sum_j \mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs} + \sum_j \mathbf{P}_{ij}^r \mathbf{C}_{ij}^r,$$

⁴Huo et al. (2021) include a discussion of capital accumulation: They show that 80 percent of the dynamic response to shocks occurs on impact. The result is important for their purpose of extracting shocks from the data; It is less important for our purpose as we are using the model to simulate empirical measures of openness.

which we can rewrite as

$$P_{i}^{r} Y_{i}^{r} = \sum_{j} P_{j}^{c} C_{j} \pi_{ij}^{r} + \sum_{j} \sum_{s} (1 - \eta^{s}) P_{j}^{s} Y_{j}^{s} \xi_{ij}^{rs},$$

where we used the facts that $P_{ij}^r C_{ij}^r = P_j^c C_j \pi_{ij}^r$ and $P_{ij}^{rs} M_{ij}^{rs} = (1 - \eta^s) P_j^s Y_j^s \xi_{ij}^{rs}$. Following Huo et al. (2021) we impose financial autarky, which implies all of value added is consumed, i.e., $P_j^c C_j = \sum_s \eta^s P_j^s Y_j^s$. Market clearing becomes

$$P_{i}^{r} Y_{i}^{r} = \sum_{j} \sum_{s} \eta^{s} P_{j}^{s} Y_{j}^{s} \pi_{ij}^{r} + \sum_{j} \sum_{s} (1 - \eta^{s}) P_{j}^{s} Y_{j}^{s} \xi_{ij}^{rs}.$$
 (1)

In deviations from the steady state, the market clearing condition in equation (1) yields an expression for prices P_i^r in terms of quantities Y_i^r . The linearized production function in which optimal labor supply and material use are substituted yields an expression for quantities Y_i^r in terms of prices P_i^r . The two equations imply a closed form solution for the equilibrium deviations of real sector output Y_i^r from the steady state. Huo et al. (2021) show that this solution is given by

$$\ln \mathbf{Y}_t = \mathbf{\Lambda}^{-1} \ln \mathbf{Z}_t,\tag{2}$$

where \mathbf{Y}_t is the vector of gross outputs Y_i^r , \mathbf{Z}_t is the vector of exogenous supply shocks Z_i^r , and In denotes deviations from the steady state, also reflected in the time subscript t. Λ is defined in Appendix A, where we also review the key steps of the derivation. Real gross output in sector (i, r) depends on the realization of shocks in all the sectors, domestic or foreign. Huo et al. (2021) label Λ^{-1} an "influence matrix" that summarizes the interdependence between sectors across countries via trade in intermediate and final goods.⁵ Λ^{-1} takes the form of a Leontief inverse, so that shocks can affect output at any order. The property extends to the response of real value added, given by

$$\ln \mathbf{V}_t = \frac{1}{\eta} \ln \mathbf{Z}_t + \alpha \ln \mathbf{H}_t,$$

With equilibrium labor, the response of value added becomes

$$\ln \mathbf{V}_t = \frac{1}{\eta} \ln \mathbf{Z}_t + \frac{\alpha \psi}{1 + \psi} \left[\ln \mathbf{P} \mathbf{Y}_t - \ln \mathbf{P}_t^c \right].$$
(3)

⁵The influence matrix was introduced by Baqaee and Farhi (2019) in a long run model of international trade.

2.2 High Order Trade

We can decompose market clearing according to border crossings:

$$\mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r} = \left[\sum_{s} \sum_{j \neq i} \mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs} + \sum_{j \neq i} \mathbf{P}_{ij}^{r} \mathbf{C}_{ij}^{r}\right] + \left[\sum_{s} \mathbf{P}_{ii}^{rs} \mathbf{M}_{ii}^{rs} + \mathbf{P}_{ii}^{r} \mathbf{C}_{ii}^{r}\right],$$

where the second term isolates a component focused on domestic uses only. Define $a_{ij}^{rs} = \frac{P_{ij}^{rs} M_{ij}^{rs}}{P_i^s Y_i^s}$ the entry in a direct requirement matrix. Iterating,

$$P_{i}^{r} Y_{i}^{r} = \left[P_{ii}^{r} C_{ii}^{r} + \sum_{s} a_{ii}^{rs} P_{ii}^{s} C_{ii}^{s} + \sum_{s} \sum_{t} a_{ii}^{rs} a_{ii}^{st} P_{ii}^{t} C_{ii}^{t} + \dots\right] \\ + \left[\sum_{j \neq i} P_{ij}^{r} C_{ij}^{r} + \sum_{s} \sum_{j \neq i} \left(a_{ij}^{rs} P_{jj}^{s} C_{jj}^{s} + a_{ii}^{rs} P_{ij}^{s} C_{ij}^{s}\right) \\ + \sum_{t} \sum_{s} \sum_{j \neq i} \left(a_{ij}^{rs} \sum_{k} a_{jk}^{st} P_{kk}^{t} C_{kk}^{t} + a_{ii}^{rs} a_{ij}^{st} P_{jj}^{t} C_{jj}^{t} + a_{ii}^{rs} a_{ii}^{st} P_{ij}^{t} C_{ij}^{t}\right) + \dots\right] \\ \equiv (P_{i}^{r} Y_{i}^{r})_{\text{DOM}} + (P_{i}^{r} Y_{i}^{r})_{\text{FOR}}$$
(4)

The first infinite sum in equation (4), denoted with $(P_i^r Y_i^r)_{DOM}$ collects all the manners in which production in sector r reaches final demand while never crossing a border, at any order. The second infinite sum $(P_i^r Y_i^r)_{FOR}$ captures all the ways in which good r in country i can cross borders to meet final demand, again at any order.

Definition 1. Define HOT_i^r by

$$HOT_i^r = \frac{(P_i^r Y_i^r)_{FOR}}{P_i^r Y_i^r}.$$
(5)

 HOT_i^r measures the fraction of production in sector r of country i whose downstream uses are located abroad.

Proposition 1. *High order trade* HOT_i^r *is the typical element of the following Hadamard division*

$$\left[(\mathbf{I} - \mathbf{A}^m)^{-1} \mathbf{P} \mathbf{C} - (\mathbf{I} - \mathbf{A}^m_{\text{DOM}})^{-1} \mathbf{P} \mathbf{C}_{\text{DOM}} \right] \oslash \left[(\mathbf{I} - \mathbf{A}^m)^{-1} \mathbf{P} \mathbf{C} \right],$$

where **PC** denotes the vector of all final demand, **PC**_{DOM} denotes final demand arising from the domestic country, \mathbf{A}^m is an NR × NR matrix with typical element a_{ij}^{rs} , and $\mathbf{A}_{\text{DOM}}^m$ is the NR × NR block-diagonal matrix with typical element a_{ii}^{rs} .

It is useful to compare the equilibrium responses of value added and HOT. From Definition 1, deviations of HOT from its steady state are given by

$$\ln \mathbf{HOT}_t = \mathbf{H_1} \odot \left(\ln \mathbf{PY}_t - \ln \mathbf{PY}_{\mathrm{DOM},t} \right),$$

where **HOT** denotes the $NR \times 1$ vector of HOT_i^r , **PY** is the $NR \times 1$ vector of nominal output $P_i^r Y_i^r$, and **PY**_{DOM} is the $NR \times 1$ vector of $(P_i^r Y_i^r)_{DOM}$. **H**₁ is a $NR \times 1$ vector with typical element $\frac{1-HOT_i^r}{HOT_i^r}$ and \odot is the Hadamard product. $\ln HOT_t$ is proportional to the response of nominal output to foreign shocks, given by $\ln PY_t - \ln PY_{DOM,t}$.

From equation (3), we also know that the response of real value added to foreign shocks is given by

$$\ln \mathbf{V}_t - \ln \mathbf{V}_{\text{DOM},t} = \frac{\alpha \psi}{1 + \psi} \left[\ln \mathbf{P} \mathbf{Y}_t - \ln \mathbf{P} \mathbf{Y}_{\text{DOM},t} - (\ln \mathbf{P}_t^c - \ln \mathbf{P}_{\text{DOM},t}^c) \right],$$

where $\ln \mathbf{P}_{\text{DOM},t}^c$ denotes the response of the consumer price index to domestic supply shocks. Assuming away any response in consumer price indices, this shows that the responses of value added and of HOT to foreign shocks are proportional: The fluctuations in HOT provide an approximation of the response of value added to foreign shocks, i.e., a measure of foreign exposure.

Of course in general the consumer price index does respond to foreign shocks, so the two responses are not proportional. In practice, the response of the CPIs depend on the elasticities of substitution in final and intermediate consumption. The quality of the approximation increases in the elasticities, since for high substitutability the responses of prices to supply shocks are muted. In addition, with high values of the elasticities positive supply shocks affect downstream demand positively since the increase in quantities is larger than the fall in prices. As a result downstream demand increases in response to upstream supply shocks with consequences throughout the network, which generalizes Acemoglu, Akcigit, and Kerr (2016).⁶

2.3 Alternative Measures

In our simulations we consider three commonly used measures of "openness". The first one is given by the ratio of direct exports to value added, used at country level in Alcalá and Ciccone (2004) or Frankel and Romer (1999) among many others. The second one is approximating trade costs on the basis of observed bilateral trade flows, often used at sector level for example by Baldwin et al. (2003) or Head and Mayer (2004). The third one exploits high order linkages to isolate the value added content of trade, and was introduced by Johnson and Noguera (2012).

With the possible exception of the first one, none of these measures was designed or theoretically motivated with the purpose of capturing foreign exposure. Our aim here is not to run a horse race, because our reading of the literature suggests there are no other available measures of foreign exposure that can account for the propagation of shocks via supply chains, that are

⁶See also Guerrieri et al. (2021).

theoretically motivated, and that are easy to measure at sector level. As a result, foreign exposure has often been approximated in an *ad hoc* manner, for example using one of the measures we discuss in this section, for lack of better alternatives. Our comparison exercise is meant to illustrate how dangerously misguided this can be in empirical work.

We now describe how the three alternative measures we consider in our simulations are derived in the model. The details of the derivations are presented in Appendix B, and make use of the definitions in Appendix A.

The ratio of direct exports to output

Total intermediate and final direct exports from country-sector (i, r) normalized by value added can be written as

$$\mathbf{X}_{i}^{r} = \frac{\sum_{j \neq i} \mathbf{P}_{ij}^{r} \mathbf{C}_{ij}^{r} + \sum_{j \neq i} \sum_{s} \mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs}}{\mathbf{P}_{i}^{r} \mathbf{V} \mathbf{A}_{i}^{r}}.$$
(6)

Using the model's notation, the steady state value of the ratio is given by

$$\begin{aligned} \mathbf{X}_{i}^{r} &= \sum_{j \neq i} \frac{a c_{ij}^{r} \mathbf{P}_{j}^{c} \mathbf{C}_{j}}{\eta^{r} \mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} + \sum_{s} \sum_{j \neq i} \frac{\mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs}}{\eta^{r} \mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} \\ &= \sum_{j \neq i} \frac{b c_{ij}^{r}}{\eta^{r}} + \sum_{s} \sum_{j \neq i} \frac{b_{ij}^{rs}}{\eta^{r}}, \end{aligned}$$

where $ac_{ij}^r = \frac{P_{ij}^r C_{ij}^r}{P_j^c C_j}$, $b_{ij}^{rs} = \frac{P_{ij}^{rs} M_{ij}^{rs}}{P_i^r Y_i^r}$, and $bc_{ij}^r = \frac{P_{ij}^r C_{ij}^r}{P_i^r Y_i^r}$. In deviations from the steady state, this implies

$$\ln \mathbf{X}_{i,t}^{r} = \frac{1}{\mathbf{X}_{i}^{r}} \left[\sum_{j \neq i} \frac{ac_{ij}^{r} \mathbf{P}_{j}^{c} \mathbf{C}_{j}}{\eta^{r} \mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} (\ln \mathbf{P}_{ij,t}^{r} \mathbf{C}_{ij,t}^{r} - \ln \mathbf{P}_{i,t}^{r} \mathbf{Y}_{i,t}^{r}) \right. \\ \left. + \sum_{s} \sum_{j \neq i} \frac{\mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs}}{\eta^{r} \mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} (\ln \mathbf{P}_{ij,t}^{rs} \mathbf{M}_{ij,t}^{rs} - \ln \mathbf{P}_{i,t}^{r} \mathbf{Y}_{i,t}^{r}) \right] \\ \left. = \frac{1}{\eta^{r}} \frac{\sum_{j \neq i} bc_{ij}^{r}}{\mathbf{X}_{i}^{r}} \ln \mathbf{P}_{ij,t}^{r} \mathbf{C}_{ij,t}^{r} + \frac{1}{\eta^{r}} \frac{\sum_{s} \sum_{j \neq i} b_{ij}^{rs}}{\mathbf{X}_{i}^{r}} \ln \mathbf{P}_{ij,t}^{rs} \mathbf{M}_{ij,t}^{rs} - \ln \mathbf{P}_{i,t}^{rs} \mathbf{Y}_{i,t}^{r}. \right.$$

In Appendix **B** we derive equilibrium expressions for $\ln P_{ij,t}^r C_{ij,t}^r$, $\ln P_{ij,t}^{rs} M_{ij,t}^{rs}$, and $\ln P_{i,t}^r Y_{i,t}^r$ that are substituted into $\ln X_{i,t}^r$ to obtain a reduced form expression.

A proxy for trade costs

Baldwin et al. (2003) and Head and Mayer (2004) introduce a measure inspired directly from the gravity model that they label the "phiness" of trade. The idea is to normalize direct bilateral trade at sector level by adequately chosen measures so that the ratio maps into trade costs in a way that is grounded in theory. They show that the cost of trading good r between country i and country j maps into

$$\phi_{ij}^{r} = \left(\frac{(\mathbf{P}_{ij}^{r}\,\mathbf{M}_{ij}^{r} + \mathbf{P}_{ij}^{r}\,\mathbf{C}_{ij}^{r}) \times (\mathbf{P}_{ji}^{r}\,\mathbf{M}_{ji}^{r} + \mathbf{P}_{ji}^{r}\,\mathbf{C}_{ji}^{r})}{(\mathbf{P}_{ii}^{r}\,\mathbf{M}_{ii}^{r} + \mathbf{P}_{ii}^{r}\,\mathbf{C}_{ii}^{r}) \times (\mathbf{P}_{jj}^{r}\,\mathbf{M}_{jj}^{r} + \mathbf{P}_{jj}^{r}\,\mathbf{C}_{jj}^{r})}\right)^{\frac{1}{2}},$$

where $P_{ij}^r M_{ij}^r = \sum_s P_{ij}^{rs} M_{ij}^{rs}$ is the total value of the intermediate sales of good r produced in country i across all sectors in country j. The denominator contains each country's "imports from itself", calculated as the value of all shipments from sector r to any sector s that remain in the producing country. The phiness of trade for sector r in country i can then be defined by an average of ϕ_{ij}^r across partner countries j:

$$\phi_i^r = \frac{1}{J} \sum_{j \neq i} \phi_{ij}^r. \tag{7}$$

Using the model's notation, the definition of ϕ_{ij}^r implies the following steady state value:

$$\begin{split} \phi_{ij}^r &= \left(\frac{\Phi_{ij}^r}{\Phi_{ii}^r} \times \frac{\Phi_{ji}^r}{\Phi_{jj}^r}\right)^{\frac{1}{2}} \\ &= \left(\frac{\sum_s b_{ij}^{rs} + bc_{ij}^r}{\sum_s b_{ii}^{rs} + bc_{ii}^r} \times \frac{\sum_s b_{ji}^{rs} + bc_{ji}^r}{\sum_s b_{jj}^{rs} + bc_{jj}^r}\right)^{\frac{1}{2}}, \end{split}$$

where $\Phi_{ij}^r = \sum_s \frac{P_{ij}^{rs} M_{ij}^{rs}}{P_i^r Y_i^r} + \frac{P_{ij}^r C_{ij}^r}{P_i^r Y_i^r}$. In deviations from the steady state

$$\ln \phi_{ij,t}^r = \frac{1}{2} (\phi_{ij}^r)^{-\frac{1}{2}} \left(\ln \Phi_{ij,t}^r - \ln \Phi_{ii,t}^r + \ln \Phi_{ji,t}^r - \ln \Phi_{jj,t}^r \right).$$

Aggregating to the country level

$$\ln \phi_{i,t}^{r} = \sum_{j \neq i} \frac{\phi_{ij}^{r}}{\phi_{i}^{r}} \ln \phi_{ij,t}^{r}$$
$$= \frac{1}{2} \sum_{j \neq i} \frac{(\phi_{ij}^{r})^{\frac{1}{2}}}{\phi_{i}^{r}} \left(\ln \Phi_{ij,t}^{r} - \ln \Phi_{ii,t}^{r} + \ln \Phi_{ji,t}^{r} - \ln \Phi_{jj,t}^{r} \right)$$

Each element $\ln \Phi_{ij,t}^r$ of $\ln \phi_{i,t}^r$ depends on $\ln P_{ij,t}^{rs} M_{ij,t}^{rs}$, $\ln P_{ij,t}^r C_{ij,t}^r$, and $\ln P_{i,t}^r Y_{i,t}^r$ whose expressions are derived in Appendix B. We use these expressions to obtain the reduced form expression for $\ln \phi_{i,t}^r$ in terms of the fundamentals of the model.

Trade in Value Added

Johnson and Noguera (2012) introduce TiVA_{i}^{r} , a measure of the value added content of exports of good r produced in country i. TiVA_{i}^{r} is defined as the typical element of the following product

$$\left(\frac{\mathbf{PVA}}{\mathbf{PY}}\right)(\mathbf{I}-\mathbf{A}^{m})^{-1} (\mathbf{PC}-\mathbf{PC}_{\mathrm{DOM}}),\tag{8}$$

where $\frac{\mathbf{PVA}}{\mathbf{PY}}$ is an NR × NR diagonal matrix with the ratio of nominal value added to gross output in sector r of country i on the diagonal.

HOT is intimately related to TiVA. Johnson (2018) writes that TiVA "decompose final goods by location of consumption", a definition that seems very close to HOT's. But there is a fundamental difference. TiVA measures the fragmentation of exports; Instead, HOT measures the fragmentation of output, the fraction of gross output that is sold across a border. This difference is apparent from the fact that HOT applies different Leontief inverses to PC and to PC_{DOM} , whereas TiVA applies the same, i.e., decomposes exports: TiVA is not a measure of foreign exposure.

HOT is bounded between 0 and 1, whereas TiVA is not: As a consequence TiVA is often normalized. Here we normalize TiVA by value added, which accounts for scale, as in Duval et al. (2016).⁷ We define

$$\mathbf{T}_{i}^{r}(\mathbf{VA}) = \frac{\mathbf{TiVA}_{i}^{r}}{\mathbf{P}_{i}^{r} \mathbf{VA}_{i}^{r}},$$

Using the model's notation, the definition of $T_i^r(VA)$ implies the following steady state value:

$$T_i^r(VA) = \sum_j \sum_s \lambda_{ij}^{rs} \frac{P_{ij}^r C_{ij}^r - P_{ii}^r C_{ii}^r}{P_i^r Y_i^r}$$
$$= \sum_j \sum_s \lambda_{ij}^{rs} (bc_{ij}^r - bc_{ii}^r),$$

where λ_{ij}^{rs} is a typical element of $(\mathbf{I} - \mathbf{A}^m)^{-1}$. In deviations from the steady state

$$\ln T_{i,t}^{r}(VA) = \frac{\sum_{j} \sum_{s} \lambda_{ij}^{rs}}{\sum_{j} \sum_{s} \lambda_{ij}^{rs} (bc_{ij}^{r} - bc_{ii}^{r})} (bc_{ij}^{r} \ln P_{ij,t}^{r} C_{ij,t}^{r} - bc_{ii}^{r} \ln P_{ii,t}^{r} C_{ii,t}^{r}) - \ln P_{i,t}^{r} Y_{i,t}^{r}.$$

 $\ln T_{i,t}^r(VA)$ depends on $\ln P_{ij,t}^r C_{ij,t}^r$ and $\ln P_{i,t}^r Y_{i,t}^r$, whose expressions are derived in Appendix **B**.

⁷Another option, followed by Johnson and Noguera (2012), is to divide TiVA by total exports, which quantifies the importance of indirect vs. direct trade. But with this normalization, TiVA takes explosive values in sectors with little direct trade.

2.4 Simulations

We exploit the model to simulate the responses to shocks of all variables of interest. Our objective is to gauge which measure(s) best replicate the simulated responses of real value added to a foreign supply shock. The responses of HOT ($\ln HOT_t$) and of value added ($\ln V_t$) are simulated using the equations obtained in Section 2.2. The responses of the alternative measures are simulated using the expressions in Section 2.3.

Calibration

Appendices A and B contain the definitions of all the variables that are necessary to compute the deviations from the steady state $\ln V_{i,t}^r$, $\ln HOT_{i,t}^r$, $\ln X_{i,t}^r$, $\ln \phi_{i,t}^r$, and $\ln T_{i,t}^r$ (VA). We now describe the data used to calibrate their constituent elements.

Define the world input-output matrix W with typical element $P_{ij}^{rs} M_{ij}^{rs}$. W contains the bulk of the information available from the world input-output database WIOD: It reports intermediate trade within and between countries, augmented with vectors of final demand $P_{ij}^r C_{ij}^r$. Final demand breaks down into a domestic and an international component by country *j*, but not by sector *s*.

In addition, W also keeps track of the net inventories INV_{ij}^r in sector r of country i, broken down by country use j, but not by sector use s. To account for inventories, we follow Antràs and Chor (2013, 2018) and correct the input-output data in WIOD according to a proportion rule. We rescale each entry $P_{ij}^{rs} M_{ij}^{rs}$ and $P_{ij}^r C_{ij}^r$ in W by $P_i^r Y_i^r / (P_i^r Y_i^r - \text{INV}_i^r)$ where $\text{INV}_i^r = \sum_j \text{INV}_{ij}^r$. We denote with W* the resulting rescaled input-output matrix.

The direct requirement matrix \mathbf{A}^m is then computed on the basis of this rescaled inputoutput matrix. The typical element of \mathbf{A}^m , a_{ij}^{rs} , is the typical element in \mathbf{W}^* normalized by the column-wise sum of its elements, i.e. sector-level gross output (corrected for inventories). To define $\mathbf{A}_{\text{DOM}}^m$ we extract the block diagonal of \mathbf{A}^m that contains the within country components of the direct requirement matrix. We also extract the domestic components of **PC** to define \mathbf{PC}_{DOM} .

The 2016 release of WIOD provides data for 43 developed and developing countries from 2000 to 2014. This represents approximately 85 percent of world GDP. The input-output data are in millions USD at current prices and are available for 56 sectors for each country and each year. We exclude 6 public sectors from our analysis.⁸

⁸See http://www.wiod.org/database/iot.html and Dietzenbacher et al. (2013) for details on the methodology used to construct these data.

Simulations

We simulate the responses of value added, HOT, X, ϕ , and T(VA) to a US supply shock. The shock is calibrated to the empirical standard deviation of aggregate gross output in the US. We collect the sector-level responses of all five variables for 42 countries, excluding the US, which implies a simulated dataset of 50 × 42 observations on which we perform regression analysis. We present the results in Table 1. Some robustness checks for different values of the elasticities ρ and ϵ are presented in Tables C.1 - C.3 available in Online Appendix C.

Table 1 presents the results of regression analysis performed on simulated data. In the main text we calibrate the model with the elasticity estimates obtained by Huo et al. (2021): $\rho = 1.43$, $\epsilon = 0.89$, and $\psi = 0.723$.⁹ The dependent variable is $\ln V$ across specifications. The regressors $\ln HOT$, $\ln X$, $\ln \phi$, and $\ln T(VA)$ are first included individually, and then included simultaneously. The simulation results are clear from Table 1: The responses of value added and HOT correlate positively and significantly. Including controls for $\ln X$, $\ln \phi$, or $\ln T(VA)$ does not alter the result. The coefficients on the alternative measures of openness are unstable and often negative and significant. These estimation results do not appear to depend on the calibration choice for ρ and ϵ , as we document in Appendix C.

3 Foreign Exposure in the Data

We now compute the various considered measures for every year available directly from the world input-output database WIOD. HOT_i^r is computed on the basis of Proposition 1. To illustrate the importance of high orders of trade, we also include a version of HOT limited to direct trade, which we label PX/PY. X_i^r is computed from its definition in equation (6), and is denoted as PX/PVA in this section to differentiate it from the first-order component of HOT. ϕ_i^r is computed from equation (7), and T_i^r (VA) is computed from its definition in equation (8). We open the section with some stylized facts about HOT and how it differs from other measures. We then move to estimations that purport to verify in the data that HOT is a good measure of foreign exposure.

⁹These are the estimates obtained using instrumental variables. They are reported in Appendix C1 and C2 of Huo et al. (2021). In Appendix C we present further regression results based on simulated data obtained with alternative values of ρ and ϵ reported in Huo et al. (2021).

3.1 Stylized facts

Table 2 reports the correlations between the five measures we consider: HOT, PX/PY, PX/PVA, ϕ , and T(VA). The highest correlation is between HOT and its first-order component PX/PY, suggesting direct trade is a driving force of HOT. HOT also correlates highly with T(VA), presumably because both incorporate high-order trade linkages. On the other hand, ϕ and PX/PVA tend to correlate weakly with the other measures.

Figure 1 reports the distributions of all five measures across sectors within each country. The ranking of countries according to HOT is not surprising in the sense that small open economies tend to have distributions centered around large median values (Ireland, Luxembourg, the Netherlands), and large economies tend to be centered around small median values (Japan, Brazil, the US). In most countries the distribution of HOT across sectors is by and large symmetric, with values that cover almost all of its support. There are open sectors in relatively closed countries: for example, HOT takes maximum values above 0.6 in some sectors in Japan. And there are closed sectors in open economies, even in Ireland or the Netherlands where minimum values for HOT are below 0.1.

The ranking of countries is different for the other measures: "open" vs. "closed" countries are not the same according to HOT, PX/PVA, ϕ , or T(VA). For example, the median sector in China is exporting a tiny percentage of its value added according to PX/PVA. And the median value of ϕ is very low in Brazil. The cross-sector distributions are also very different across measures. While HOT is distributed broadly over its support, PX/PVA, ϕ , and T(VA) are heavily skewed to the right, with mostly small and a few large values. For example open countries according to PX/PVA (Belgium, Ireland, or Luxembourg) are characterized by very few open sectors and a majority that are relatively closed. This is very different from what HOT implies, i.e., most, if not all sectors in those countries are in fact (indirectly) exposed to foreign developments.

Figure 2 reports the distributions of all five measures across countries for each sector. Two conclusions emerge. Firstly, the ranking of sectors according to HOT is intuitive: according to median HOT, services (Construction, Trade) are relatively "closed" while manufacturing sectors (Metals, Machinery, Chemicals) are relatively "open". The ranking of sectors is sometimes less intuitive for other measures. Secondly, no sector is fully insulated from foreign developments according to HOT, even putative "non traded" sectors. For example, there are countries in which indirect exports of IT represent just shy of 100 percent of output. Some countries also export indirectly up to 20 percent of output in Real Estate. This is absolutely not the case for the other measures, e.g., PX/PVA, ϕ , and T(VA) that instead suggest some sectors are in fact non traded in almost all countries.

In both Figures, the comparison between HOT and its first-order component PX/PY is inter-

esting. On the one hand, the country and sector rankings are similar according to both measures, a reflection of the high correlation in Table 2. On the other hand, the distributions are quite different. In Figure 1, "closed" countries display much more concentrated distributions in PX/PY than in HOT, meaning that direct trade is limited for most sectors in those countries. The same feature is apparent from Figure 2: "closed" sectors have very concentrated distributions according to PX/PY, i.e., direct trade is very limited in those sectors. Both differences are meaningful when it comes to exploring empirically what measure best captures foreign exposure.

3.2 Estimations

The simulations in Section 2.4 demonstrate that HOT performs best among openness measures at replicating the consequences of foreign supply shocks on output. We now examine whether this is also true empirically. The shocks are well identified in the simulation but not in the data, where many are likely to occur simultaneously in many locations. For this reason in the empirics we consider the average properties of HOT, PX/PY, PX/PVA, ϕ , and T(VA) as defined by their model-implied steady state values introduced in Sections 2.2 and 2.3.

As in the model of Section 2.1, a good measure of foreign exposure should isolate the short and long-run responses of activity to foreign developments. We investigate this empirically with simple regression analysis. Firstly we examine the correlates of fluctuations in value added at country-sector level. Secondly we extend the analysis to a bilateral environment and examine the correlates of the synchronization in the fluctuations of value added, again at country-sector level. Thirdly we examine the correlates of country-sector long run growth. In each case, the set of correlates includes each of the five measures considered in this section, individually and all at once.¹⁰

Fluctuations in Value Added

We investigate whether fluctuations in value added are correlated with any of the measures of exposure we consider in this paper. We estimate:

$$\ln \mathbf{V}_{i,t}^r = \alpha + \beta_1 \operatorname{HOT}_{i,t}^r + \beta_2 (\operatorname{PX}/\operatorname{PY})_{i,t}^r + \beta_3 (\operatorname{PX}/\operatorname{PVA})_{i,t}^r + \beta_4 \phi_{i,t}^r + \beta_5 \operatorname{T}_{i,t}^r (\operatorname{VA}) + \varepsilon_{i,t}^r,$$
(9)

where $\ln V_{i,t}^r$ denotes the fluctuations in real value added in country-sector (i, r), measured as the cyclical component of value added implied by the Hodrick-Prescott (HP) filter, or alternatively its growth rate. The specification is akin to Alcalá and Ciccone (2004) but in a panel of country-sectors while they worked on a cross section of countries. It is also related to a large

¹⁰Real value added, synchronization, and growth are obtained from the Socio-Economic Accounts available from the 2016 release of WIOD. Nominal values are deflated using the sector-level price indices from the same source. Detail on the computation of all variables can be found in Appendix D

literature correlating firm-level production with its export status.¹¹

Table 3 presents the results. The upper panel uses the HP filter to isolate the cyclical component of value added, the lower panel computes growth rates. In both panels HOT is positive and significant whether it is included alone or along with other measures, which suggests HOT captures the component of value added fluctuations that originates from foreign shocks. None of the other considered measure performs well: first-order trade PX/PY is insignificant, while direct trade, trade costs, and TiVA display unstable estimates.

We classify the 50 sectors in WIOD into Agriculture, Manufacturing, and Services and perform estimation (9) within each of the three categories in Table 4. We report results when all measures are included simultaneously to save on space. HOT continues to be the only measure that correlates positively with value added across all three categories, including services. This is consistent with HOT capturing the foreign exposure of sectors that are customarily considered non traded.

Synchronization

We explore the correlation between synchronization and openness by estimating

$$SYNC_{ij,t}^{rs} = \alpha_{ij}^{rs} + \gamma_t + \beta_1 \operatorname{HOT}_{ij,t}^{rs} + \beta_2 (\operatorname{PX}/\operatorname{PY})_{ij,t}^{rs} + \beta_3 (\operatorname{PX}/\operatorname{PVA})_{ij,t}^{rs} + \beta_4 \phi_{ij,t}^{rs} + \beta_5 \operatorname{T}_{ij,t}^{rs} (\operatorname{VA}) + \varepsilon_{ij,t}^{rs},$$
(10)

where SYNC^{*rs*}_{*ij,t*} denotes the correlation between fluctuations in country-sectors (i, r) and (j, s) at time *t*, and the other regressors are bilateral versions of the measures considered up to now. It is important to estimate equation (10) within panel, see Kalemli-Özcan et al. (2013), which is the reason why fixed effects are included. The fixed effects here are very general and specific to each country-sector pair (i, j, r, s).

A popular measure of $SYNC_{ij,t}^{rs}$ computes the quasi correlation between sector growth rates, defined by

$$\mathbf{SYNC}_{ij,t}^{rs} = rac{(g_{i,t}^r - \bar{g}_i^r) imes (g_{j,t}^s - \bar{g}_j^s)}{\sigma_i^r \sigma_j^s},$$

where \bar{g}_i^r and σ_i^r denote the mean and standard deviation of $g_{i,t}^r$. The measure was implemented among others in Duval et al. (2016).

 $HOT_{ij,t}^{rs}$ is defined to reflect how much two sectors are open to each other, and also how much they are each open to foreign shocks happening in third countries, at any order through the supply chain. Given the theoretical mapping between the response of value added to foreign

¹¹See for example Bernard and Jensen (1995, 1999, 2004), Amiti and Konings (2007), Topalova and Khandelwal (2011), Bernard et al. (2018), or De Loecker and Van Biesebroeck (2018)

shocks and HOT, it is natural to compute a pairwise product between foreign exposures in country-sectors (i, r) and (j, s).¹² Define

$$\operatorname{HOT}_{ij,t}^{rs} = \operatorname{HOT}_{i,t}^{r} \times \operatorname{HOT}_{j,t}^{s},$$

which conflates bilateral and multilateral sources of co-movements. $(PX / PY)_{ij,t}^{rs}$ is defined as the first-order component of $HOT_{ij,t}^{rs}$.

It is straightforward to extend the other three measures of openness to a bilateral context. Since bilateral trade data are typically only available for intermediate goods, we define

$$(\mathrm{PX} / \mathrm{PVA})_{ij,t}^{rs} = \left(\frac{\mathrm{PM}_{ij,t}^{rs} + \mathrm{PM}_{ji,t}^{rs}}{\mathrm{PVA}_{i,t}^{r} + \mathrm{PVA}_{j,t}^{r}}\right)$$

$$\phi_{ij,t}^{rs} = \left(\frac{\mathrm{PM}_{ij,t}^{rs} \times \mathrm{PM}_{ji,t}^{rs}}{\mathrm{PM}_{ii,t}^{rs} \times \mathrm{PM}_{jj,t}^{rs}}\right)^{1/2}$$

and

$$\mathbf{T}_{ij,t}^{rs}(\mathbf{VA}) = \mathbf{T}_{i,t}^{r}(\mathbf{VA}) \times \mathbf{T}_{j,t}^{s}(\mathbf{VA})$$

by analogy with $HOT_{ij,t}^{rs}$.¹³

The specification in equation (10) generalizes a large literature that has established the correlation between bilateral trade and cycle synchronization across countries, see among many others Frankel and Rose, 1998 and Kalemli-Özcan et al., 2013). It is also related to a large literature in firm-level data.¹⁴ The only contribution at sector level is di Giovanni and Levchenko (2010) who show that synchronization between sectors increases with direct intermediate trade in the US.

The first panel of Table 5 reports the estimates of equation (10) including each regressor individually and then simultaneously. HOT is the only variable whose correlation with cycle synchronization is unanimously positive and significant. Interestingly, specification (6) shows that this result comes from high-order linkages, since the first-order component of HOT has a negative sign in the regression. The coefficients on the other measures either are unstable or have the wrong sign.

The lower panel of Table 5 introduces the six pairwise correlations between the three broad categories that are Agriculture, Manufacturing, and Services. Here the evidence is more mixed:

 $^{^{12}}$ An intuitive alternative would be to alter HOT's definition and preserve its bilateral dimension by considering final demand in specific destination countries *j*. But doing so would focus the measure on exposure to each other and exclude common exposure to third party shocks.

¹³TiVA is computed analogously to HOT, but direct trade PX/PVA and ϕ use instead the bilateral dimension of the data. This is in reference to the measures of direct trade used in the literature, starting with Frankel and Rose (1998).

¹⁴See di Giovanni et al., 2017, 2018.

HOT does not capture the exposure of Agricultural sectors as well as TiVA, but it does capture very well the exposure of Services, better than any other measures in the table. Measures of exposure based on direct trade (PX/PVA and ϕ) do not perform at all, which explains why extending the literature pioneered by Frankel and Rose (1998) has proved elusive so far.

Long Run Growth

In the long run, (foreign) supply shocks should have observable consequences on growth, which a good measure of foreign exposure should capture. We estimate

$$\Delta \ln \mathbf{V}_{i,\varsigma}^r = \alpha_r + \alpha_i + \beta_0 \, \ln \mathbf{V}_{i,\varsigma}^r + \beta_1 \, \mathrm{HOT}_{i,\varsigma}^r + \beta_2 (PX/PY)_{i,\varsigma}^r + \beta_3 (PX/PVA)_{i,\varsigma}^r \quad (11)$$
$$+ \beta_4 \, \phi_{i,\varsigma}^r + \beta_5 \, \mathrm{T}_{i,\varsigma}^r (\mathrm{VA}) + \varepsilon_{i,\varsigma}^r,$$

where ς denotes the period over which growth rates are computed and $V_{i,\varsigma}^r$ is value added at the beginning of period ς . The estimation follows Rodrik (2013), extended to include services and both country and sector fixed effects.

The existence of a relation between foreign exposure and growth is well established at firm level (see for instance Amiti and Konings (2007), Halpern et al. (2015) or Bøler et al. (2015)). It is more controversial at country level, see for example the debates between Frankel and Romer (1999) and Rodríguez and Rodrik (2000) or the survey by Baldwin (2004). Our purpose here is not to settle the question using sector level information: For one thing, we are not trying to establish any form of causality since a positive and significant estimate of β_1 in equation (11) could just mean that foreign exposure rises in fast growing countries. The purpose of equation (11) is to compare the ability of different measures to capture a correlation between long run growth and foreign exposure.

Table 6 presents the estimation results. The upper panel introduces the regressors individually and then all at once. The correlation between long run growth and HOT is positive and significant; the correlation with other measures of foreign exposure is either zero or negative. Specification (6) demonstrates that it is high-order trade that explains this positive relation, since PX/PY enters with a negative coefficient. The lower panel decomposes growth into agriculture, manufacturing, and services: HOT correlates positively with growth in manufacturing and services, and it is in fact the only measure that does.¹⁵

¹⁵Interestingly conditional convergence holds across all sector categories, notably in services, and with both country and sector fixed effects, which generalizes Rodrik (2013).

4 Conclusion

We propose a new measure of foreign exposure grounded in theory and easy to compute from standard data. In a multi-sector multi-country canonical model we show that the response of value added to foreign shocks can be approximated by the fraction of domestically produced goods sold to final consumers abroad, directly *or indirectly*. Model simulations show that this ratio performs much better than some prominent, yet largely *ad hoc* alternative measures of foreign exposure.

We compute the ratio for a large cross-section of country-sectors using standard global input-output data: The data suggests that foreign exposure is distributed much more uniformly across sectors than hitherto assumed. In particular, no sector, even those customarily catego-rized as "non traded", is insulated from foreign developments, a reflection of the globalization of supply chains. Unlike any of its main alternatives, our new measure correlates significantly with economic fluctuations, cycle synchronization, and long run growth. We interpret this according to the model's result: Our measure is particularly well-suited to capture the domestic consequences of foreign shocks.

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	(1)	(2)	(3)	(4)	(5)
ln HOT	0.018***				0.007***
	(0.001)				(0.001)
$\ln X$		0.106***			0.002
		(0.008)			(0.006)
$\ln \phi$			-0.033***		-0.007
,			(0.009)		(0.025)
$\ln T(VA)$				-0.216***	-0.205***
				(0.005)	(0.005)
ρ	1.43	1.43	1.43	1.43	1.43
ϵ	0.89	0.89	0.89	0.89	0.89
ψ	0.723	0.723	0.723	0.723	0.723
Obs.	2,002	2,000	2,018	2,018	1,987

Table 1: Simulations

Table 2: Correlations

	HOT	PX/PY	PX/PVA	ϕ	T(VA)
HOT	1				
PX/PY	0.918	1			
PX/PVA	0.153	0.163	1		
ϕ	0.062	0.077	0.013	1	
T(VA)	0.417	0.406	0.493	0.031	1

Note: The table reports the Pearson correlation coefficients between different measures of foreign exposure to shocks.

Note: The dependent variable is simulated $\ln V_{i,t}^r$. All the regressors are defined in the text. Standard errors in parentheses. The coefficient of $\ln \phi$ is multiplied by 10^4 for legibility.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
			HP Filtered	Estimatio	ns		
НОТ	0.003** (0.001)					0.007** (0.003)	0.016** (0.007)
PX/PY		0.002 (0.002)				-0.005 (0.004)	
PX/PVA			-0.003* (0.002)				-0.003 (0.002)
ϕ				0.002 (0.002)			0.002 (0.002)
T(VA)					-0.028** (0.013)		-0.003 (0.015)
Obs.	30,984	30,984	30,990	30,990	30,984	30,984	30,984
		Fi	rst Differen	ce Estimat	ions		
НОТ РХ/РҮ	0.121** (0.049)	0.038				0.357*** (0.083) -0.234***	0.194*** (0.060)
PX/PVA		(0.046)	-0.010*** (0.003)			(0.082)	-0.009** (0.004)
ϕ				-0.003* (0.002)	0.010***		-0.004** (0.002)
1(VA)					-0.210^{***} (0.045)		-0.039 (0.077)

Table 3: Fluctuations in Value Added

Note: The dependent variable is the logarithm of real value added in PPP USD. The coefficient and standard errors of ϕ are multiplied by 10^8 for legibility. Robust standard errors in parentheses, clustered at country-sector level.

	AGR	MFG	SER
	HP F	iltered Estin	nations
НОТ	0.030*	0.022***	0.090**
	(0.016)	(0.006)	(0.036)
PX/PVA	-0.023	-0.002**	-0.013***
	(0.019)	(0.001)	(0.001)
ϕ	0.002*	0.001	0.009**
7	(0.001)	(0.002)	(0.004)
T(VA)	0.013	-0.011**	-0.123
	(0.028)	(0.006)	(0.078)
Observations	1,875	11,980	14,009
	First Di	ifference Est	imations
НОТ	0.287*	0.365***	0.135*
	(0.161)	(0.075)	(0.080)
PX/PVA	-0.143**	-0.004**	-0.015***
	(0.067)	(0.002)	(0.001)
ϕ	0.004	-0.0004	-0.006***
1	(0.004)	(0.002)	(0.001)
T(VA)	-0.024	-0.100***	-0.447***
	(0.142)	(0.034)	(0.026)
Obs	1,750	11,179	13,075

Table 4: Fluctuations in Value Added: Sector breakdown

Note: The dependent variable is the logarithm of real value added in PPP USD. The coefficient and standard errors of ϕ are multiplied by 10^8 for legibility, except those in the Agriculture (ARG) sample. Robust standard errors in parentheses, clustered at country-sector level.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
НОТ	0.575*** (0.062)					0.661*** (0.062)	1.358*** (0.081)
PX/PY		-0.448*** (0.018)				-0.476*** (0.018)	
PX/PVA			-0.486*** (0.018)				-0.725*** (0.031)
ϕ				-0.185*** (0.022)			0.285*** (0.031)
T(VA)					-0.412*** (0.033)		-0.697*** (0.044)
Obs.	27,204,537	28,846,100	28,076,161	24,878,666	27,185,049	26,917,089	23,309,841
	Agr-Agr	Agr-Mfg	Agr-Ser	Mfg-Mfg	Mfg-Ser	Ser-Ser	
НОТ	0.054 (1.162)	-0.731** (0.345)	-0.087 (0.332)	1.743*** (0.218)	2.093*** (0.140)	1.813*** (0.180)	
PX/PVA	-0.113 (0.358)	0.003 (0.132)	-0.046 (0.116)	-1.243*** (0.092)	-0.656*** (0.054)	-0.464*** (0.067)	
ϕ	-0.033 (0.346)	0.498*** (0.122)	0.167 (0.105)	0.718*** (0.088)	0.219*** (0.055)	-0.109 (0.067)	
T(VA)	0.643* (0.342)	0.449*** (0.133)	0.420*** (0.144)	-1.350*** (0.114)	-0.964*** (0.080)	-0.901*** (0.111)	
Obs.	71.763	1.021.701	1.046.492	3.836.739	8.287.170	4.654.248	

Table 5: Synchronization

Note: The regressions are performed with <u>reghdfe</u> in STATA, which allows for multiple level fixed effects (see Correia, 2017). Estimations include (i, j, r, s) fixed effects and year effects. All regressors enter in logarithms. Robust standard errors in parentheses, clustered at country-sector pair level. All coefficients and standard errors have been multiplied by 10^2 for legibility.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Initial V.A.	-0.018*** (0.002)	-0.017*** (0.002)	-0.017*** (0.002)	-0.017*** (0.002)	-0.017*** (0.002)	-0.018*** (0.002)	-0.019*** (0.002)
НОТ	0.048*** (0.007)					0.085*** (0.015)	0.062*** (0.008)
PX/PY		0.030*** (0.007)				-0.039*** (0.014)	
PX/PVA			-0.001*** (0.000)				-0.002*** (0.000)
ϕ				0.006 (0.007)			-0.003 (0.008)
T(VA)					-0.014*** (0.005)		-0.019*** (0.007)
Obs.	2,066	2,066	2,066	2,066	2,066	2,066	2,066
		AGR		MFG		SER	
Initial V.A.		-0.010* (0.005)		-0.016*** (0.002)		-0.027*** (0.003)	
НОТ		-0.007		0.078***		0.053**	
		(0.047)		(0.013)		(0.024)	
PX/PVA		0.024 (0.030)		-0.002*** (0.000)		0.001 (0.002)	
ϕ		-0.005		-0.012**		0.022	
		(0.003)		(0.006)		(0.021)	
TiVA (VA)		-0.052		-0.013***		-0.062	
		(0.055)		(0.004)		(0.052)	
Observations		125		799		934	

Table 6: Growth

Note: The dependent variable is the growth of real value added and Initial V.A. denotes its initial value, both in PPP USD. All variables are averaged over the whole sample period. All regressions include sector and country fixed effects. Robust standard errors in parentheses, clustered at country-sector level. Coefficients and standard errors for ϕ_i^r have been multiplied by 10^8 for legibility, except those in the Agriculture (ARG) sample.



Figure 1: Dispersion of HOT_i^r , X_i^r / PY_i^r , X_i^r / PVA_i^r , ϕ_i^r , and T_i^r (VA) across sectors for each country in 2014. The mid-point is the median, the thick segment is the interquartile range, and the whiskers are extreme values.



Figure 2: Dispersion of HOT_i^r , X_i^r / PY_i^r , X_i^r / PVA_i^r , and $T_i^r(VA)$ across countries for each sector in 2014. The mid-point is the median, the thick segment is the interquartile range, and the whiskers are extreme values.

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Appendix A

This appendix summarizes the key steps in the derivation of the influence matrix from Huo et al. (2021). All equilibrium conditions are expressed in deviations from the steady state, denoted with time subscripts and ln-deviations. Market clearing becomes

$$\ln \mathbf{P}_{i,t}^{r} + \ln \mathbf{Y}_{i,t}^{r} = \sum_{j} \sum_{s} \frac{ac_{ij}^{r} \mathbf{P}_{j}^{c} \mathbf{C}_{j}}{\mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} \frac{\eta^{s} \mathbf{P}_{j}^{s} \mathbf{Y}_{j}^{s}}{\mathbf{P}_{j}^{c} \mathbf{C}_{j}} (\ln \mathbf{P}_{j,t}^{s} + \ln \mathbf{Y}_{j,t}^{s} + \ln \pi_{ij,t}^{r}) + \sum_{j} \sum_{s} \frac{\mathbf{P}_{j}^{s} \mathbf{Y}_{j}^{s} a_{ij}^{rs}}{\mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r}} (\ln \mathbf{P}_{j,t}^{s} + \ln \mathbf{Y}_{j,t}^{s} + \ln \xi_{ij,t}^{rs}),$$

where in addition

$$\ln \pi_{ij,t}^{r} = (1-\rho) \sum_{k,l} ac_{kj}^{l} (\ln \mathbf{P}_{i,t}^{r} - \ln \mathbf{P}_{k,t}^{l}),$$
$$\ln \xi_{ij,t}^{rs} = (1-\epsilon) \sum_{k,l} \frac{a_{kj}^{ls}}{1-\eta^{s}} (\ln \mathbf{P}_{i,t}^{r} - \ln \mathbf{P}_{k,t}^{l}).$$

We now introduce matrices of relevant steady state ratios that help define the equilibrium.

Definitions.

 \mathbf{A}^{m} is the matrix with typical element the direct requirement coefficient $a_{ij}^{rs} = \frac{\mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs}}{\mathbf{P}_{j}^{s} \mathbf{N}_{j}^{s}} = (1 - \eta^{s}) \frac{\mathbf{P}_{ij}^{rs} \mathbf{M}_{ij}^{rs}}{\mathbf{P}_{j}^{s} \mathbf{M}_{j}^{s}}$ the share of output in (j, s) that is produced using intermediate inputs from (i, r).

 \mathbf{A}^{c} is the matrix with typical element $ac_{ij}^{r} = \frac{\mathbf{P}_{ij}^{r} \mathbf{C}_{ij}^{r}}{\mathbf{P}_{j}^{c} \mathbf{C}_{j}}$ the expenditure share of country j's final consumption that is spent on final goods produced in (i, r).

B^m is the matrix with typical element the allocation coefficient $b_{ij}^{rs} = \frac{(1-\eta^s) P_j^s Y_j^s \xi_{ij}^{rs}}{P_i^r Y_i^r} = \frac{P_{ij}^{rs} M_{ij}^{rs}}{P_i^r Y_i^r}$ the share of output in source sector (i, r) that is used as intermediate input in (j, s).

B^c is the matrix with typical element $bc_{ij}^r = \frac{\pi_{ij}^r \mathbf{P}_j^c \mathbf{C}_j}{\mathbf{P}_i^r \mathbf{Y}_i^r} = \frac{\mathbf{P}_{ij}^r \mathbf{C}_{ij}^r}{\mathbf{P}_i^r \mathbf{Y}_i^r}$ the share of output in source sector (i, r) used as final consumption in country j.

 Υ is the matrix with typical element $v_i^r = \frac{\eta^r P_i^r Y_i^r}{P_i^c C_i}$ the share of nominal value added in (i, r) in total nominal consumption in country *i*.

Rewriting the resource constraint in matrix algebra making use of these definitions yields

$$\ln \mathbf{P}_{t} + \ln \mathbf{Y}_{t} = (\mathbf{B}^{c} \mathbf{\Upsilon} + \mathbf{B}^{m})(\ln \mathbf{P}_{t} + \ln \mathbf{Y}_{t}) + (1 - \rho) \left[\operatorname{diag}(\mathbf{B}^{c} \mathbf{1}) - \mathbf{B}^{c}(\mathbf{A}^{c})^{\top} \right] \ln \mathbf{P}_{t} + (1 - \epsilon) \left[\operatorname{diag}(\mathbf{B}^{m} \mathbf{1}) - \mathbf{B}^{m}(\mathbf{I} - \boldsymbol{\eta})^{-1}(\mathbf{A}^{m})^{\top} \right] \ln \mathbf{P}_{t},$$
(A.12)

which implies an equilibrium relation between prices and quantities. In deviations from the steady state, the production function can be rewritten as

$$\ln \mathbf{Y}_t = \ln \mathbf{Z}_t + \boldsymbol{\eta} \boldsymbol{\alpha} \ln \mathbf{H}_t + (\mathbf{I} - \boldsymbol{\eta}) \ln \mathbf{M}_t.$$
(A.13)

Equilibrium labor input is given by

$$\ln \mathbf{H}_t = \frac{\psi}{1+\psi} \ln \mathbf{Y}_t + \frac{\psi}{1+\psi} (\mathbf{I} - (\mathbf{A}^c)^\top \otimes \mathbf{1}) \ln \mathbf{P}_t,$$
(A.14)

where $\ln \mathbf{P}_t^c = [(\mathbf{A}^c)^\top \otimes \mathbf{1}] \ln \mathbf{P}_t$. Market clearing in the intermediate input market implies

$$\ln \mathbf{M}_t = \ln \mathbf{Y}_t + \left(\mathbf{I} - (\mathbf{I} - \boldsymbol{\eta})^{-1} (\mathbf{A}^m)^{\top} \right) \ln \mathbf{P}_t.$$
(A.15)

Combining equations (A.12)-(A.13)-(A.14)-(A.15) yields the expression for the response of real output $\ln \mathbf{Y}_t$ in the text, where we define:

$$\begin{split} \mathbf{\Lambda} &= \bigg[\mathbf{I} - \frac{\psi}{1+\psi} \boldsymbol{\eta} \boldsymbol{\alpha} \bigg(\mathbf{I} + \bigg(\mathbf{I} - (\mathbf{A}^c)^\top \otimes \mathbf{1} \bigg) \mathcal{P} \bigg) - (\mathbf{I} - \boldsymbol{\eta}) \bigg(\mathbf{I} + \bigg(\mathbf{I} - (\mathbf{I} - \boldsymbol{\eta})^{-1} (\mathbf{A}^m)^\top \bigg) \mathcal{P} \bigg) \bigg], \\ & \mathcal{P} = - \bigg(\mathbf{I} - \mathcal{M} \bigg)^+ \bigg(\mathbf{I} - \mathbf{B}^c \boldsymbol{\Upsilon} - \mathbf{B}^m \bigg), \end{split}$$

and

$$\mathcal{M} = \mathbf{B}^{c} \mathbf{\Upsilon} + \mathbf{B}^{m} + (1-\rho) \left(\operatorname{diag}(\mathbf{B}^{c} \mathbf{1}) - \mathbf{B}^{c}(\mathbf{A}^{c})^{\top} \right) + (1-\epsilon) \left(\operatorname{diag}(\mathbf{B}^{m} \mathbf{1}) - \mathbf{B}^{m}(\mathbf{I} - \boldsymbol{\eta})^{-1}(\mathbf{A}^{m})^{\top} \right).$$

The + sign stands for the Moore-Penrose inverse as I - M is not invertible. See Huo et al. (2021).

Appendix B

This appendix derives the expressions needed to characterize the responses of all four measures of openness to supply shocks. We start with the responses of $\ln P_{i,t}^r Y_{i,t}^r$, $\ln P_{ij,t}^{rs} M_{ij,t}^{rs}$, and $\ln P_{ij,t}^r C_{ij,t}^r$, and then turn to the expressions for the four measures of openness in terms of the fundamentals of the model.

Combining equations (A.12) and the reduced form expression for real output yields the response of prices to supply shocks:

$$\ln \mathbf{P}_t = \mathcal{P} \ln \mathbf{Y}_t$$

It follows the response of nominal output is given by

$$\ln \mathbf{P}\mathbf{Y}_t = (\mathcal{P} + \mathbf{I})\mathbf{\Lambda}^{-1}\ln \mathbf{Z}_t$$

From the production function, it is immediate that

$$\ln \mathbf{P}\mathbf{M}_t = \ln \mathbf{P}\mathbf{Y}_t = (\mathcal{P} + \mathbf{I})\mathbf{\Lambda}^{-1}\ln \mathbf{Z}_t.$$

This characterizes the NR ×1 vector of the responses of nominal intermediate input, with element $\ln P_{i,t}^r M_{i,t}^r$. Furthermore, in equilibrium,

$$\mathbf{P}_{ji}^{sr} \mathbf{M}_{ji}^{sr} = \xi_{ji}^{sr} \mathbf{P}_i^r \mathbf{M}_i^r.$$

It follows that in deviations from the steady state,

$$\ln \mathbf{P}_{ji,t}^{sr} \mathbf{M}_{ji,t}^{sr} = \ln \xi_{ji,t}^{sr} + \ln \mathbf{P}_{i,t}^{r} \mathbf{M}_{i,t}^{r}$$

$$= (1 - \epsilon) \sum_{k,l} \frac{a_{kj}^{ls}}{1 - \eta^{r}} (\ln \mathbf{P}_{j,t}^{s} - \ln \mathbf{P}_{k,t}^{l}) + \ln \mathbf{P}_{i,t}^{r} \mathbf{M}_{i,t}^{r},$$

which, along with the equations for $\ln P_{i,t}^r M_{i,t}^r$ and $\ln P_{j,t}^s$ completes the characterization of $\ln P_{ji,t}^{sr} M_{ji,t}^{sr}$ and $\ln P_{jj,t}^{sr} M_{jj,t}^{sr}$.

With financial autarky, nominal final expenditures in deviations from the steady state are

given by

$$\ln \mathbf{P}_{i,t}^{c} \mathbf{C}_{i,t} = \frac{\sum_{r} \eta^{r} \mathbf{P}_{i}^{r} \mathbf{Y}_{i}^{r} \ln \mathbf{P}_{i,t}^{r} \mathbf{Y}_{i,t}^{r}}{\mathbf{P}_{i}^{c} \mathbf{C}_{i}}$$
$$= \sum_{r} \upsilon_{i}^{r} \ln \mathbf{P}_{i,t}^{r} \mathbf{Y}_{i,t}^{r},$$

where v_i^r is the typical element of Υ . Furthermore, in equilibrium

$$\mathbf{P}_{ji}^r \mathbf{C}_{ji}^r = \pi_{ji}^r \mathbf{P}_i^c \mathbf{C}_i,$$

so that in deviations from the steady state,

$$\begin{split} \ln \mathbf{P}_{ji,t}^{r} \, \mathbf{C}_{ji,t}^{r} &= \ln \pi_{ji,t}^{r} + \ln \mathbf{P}_{i,t}^{c} \, \mathbf{C}_{i,t} \\ &= (1-\rho) \sum_{k,l} a c_{kj}^{l} (\ln \mathbf{P}_{j,t}^{r} - \ln \mathbf{P}_{k,t}^{l}) + \ln \mathbf{P}_{i,t}^{c} \, \mathbf{C}_{i,t}, \end{split}$$

which, along with the equations for $\ln P_{i,t}^c C_{i,t}$ and $\ln P_{j,t}^r$ completes the derivation of $\ln P_{ji,t}^r C_{ji,t}^r$ and $\ln P_{jj,t}^r C_{jj,t}^r$.

We can now express our measures of openness in terms of the fundamentals of the model. In deviations from the steady state, gross exports are given by

$$\ln \mathbf{X}_{i,t}^{r} = \frac{1}{\eta^{r} \mathbf{P}_{i}^{r} \mathbf{X}_{i}^{r}} \left[\sum_{s} \sum_{j \neq i} b_{ij}^{rs} (\ln \xi_{ij,t}^{rs} + \ln \mathbf{P}_{j,t}^{s} \mathbf{M}_{j,t}^{s}) + \sum_{j \neq i} bc_{ij}^{r} (\ln \pi_{ij,t}^{r} + \ln \mathbf{P}_{j,t}^{c} \mathbf{C}_{j,t}) \right] - \ln \mathbf{P}_{i,t}^{r} \mathbf{Y}_{i,t}^{r}$$

In deviations from the steady state the phiness of trade is given by

$$\begin{split} \ln \phi_{i,t}^{r} &= \frac{1}{2} \sum_{j \neq i} \frac{(\phi_{ij}^{r})^{\frac{1}{2}}}{\phi_{i}^{r}} \left(\ln \Phi_{ij,t}^{r} - \ln \Phi_{ii,t}^{r} + \ln \Phi_{ji,t}^{r} - \ln \Phi_{jj,t}^{r} \right) \\ &= \frac{1}{2} \sum_{j \neq i} \frac{(\phi_{ij}^{r})^{\frac{1}{2}}}{\phi_{i}^{r}} \left[\frac{\sum_{s} b_{ij}^{rs}}{\sum_{s} b_{ij}^{rs} + bc_{ij}^{r}} (\ln \xi_{ij,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) + \frac{bc_{ij}^{r}}{\sum_{s} b_{ij}^{rs} + bc_{ij}^{r}} (\ln \pi_{ij,t}^{r} + \ln P_{j,t}^{s} C_{j,t}) \right. \\ &- \frac{\sum_{s} b_{ii}^{rs}}{\sum_{s} b_{ii}^{rs} + bc_{ii}^{r}} (\ln \xi_{ii,t}^{rs} + \ln P_{i,t}^{s} M_{i,t}^{s}) - \frac{bc_{ii}^{r}}{\sum_{s} b_{ii}^{rs} + bc_{ii}^{r}} (\ln \pi_{ii,t}^{r} + \ln P_{i,t}^{c} C_{i,t}) \\ &+ \frac{\sum_{s} b_{ji}^{rs}}{\sum_{s} b_{ji}^{rs} + bc_{ji}^{r}} (\ln \xi_{ji,t}^{rs} + \ln P_{i,t}^{s} M_{i,t}^{s}) + \frac{bc_{ji}^{r}}{\sum_{s} b_{ji}^{rs} + bc_{ji}^{r}} (\ln \pi_{ji,t}^{r} + \ln P_{i,t}^{c} C_{i,t}) \\ &- \frac{\sum_{s} b_{jj}^{rs}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{ji}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{i,t}^{c} C_{j,t}) \\ &- \frac{\sum_{s} b_{jj}^{rs}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{j,t}^{c} C_{j,t}) \\ &- \frac{\sum_{s} b_{jj}^{rs}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{j,t}^{c} C_{j,t}) \\ &- \frac{\sum_{s} b_{jj}^{rs}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{j,t}^{c} C_{j,t}) \\ &- \frac{\sum_{s} b_{jj}^{rs}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{j,t}^{c} C_{j,t}) \\ \\ &- \frac{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) - \frac{bc_{jj}^{r}}{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \pi_{jj,t}^{r} + \ln P_{j,t}^{s} C_{j,t}) \\ \\ &- \frac{\sum_{s} b_{jj}^{rs} + bc_{jj}^{r}} (\ln \xi_{jj,t}^{rs} + \ln P_{j,t}^{s} M_{j,t}^{s}) -$$

In deviations from the steady state, $\mathrm{T}(\mathrm{VA})$ can be written as

$$\ln T_{i,t}^{r}(VA) = \frac{\sum_{j} \sum_{s} \lambda_{ij}^{rs}}{\sum_{j} \sum_{s} \lambda_{ij}^{rs} (bc_{ij}^{r} - bc_{ii}^{r})} (bc_{ij}^{r} \ln P_{ij,t}^{r} C_{ij,t}^{r} - bc_{ii}^{r} \ln P_{ii,t}^{r} C_{ii,t}^{r}) - \ln P_{i,t}^{r} Y_{i,t}^{r}$$
$$= \frac{\sum_{j} \sum_{s} \lambda_{ij}^{rs} bc_{ij}^{r}}{\sum_{j} \sum_{s} \lambda_{ij}^{rs} (bc_{ij}^{r} - bc_{ii}^{r})} (\ln P_{j,t}^{r} C_{j,t}^{r} + \ln \pi_{ij,t}^{r})$$
$$- \frac{\sum_{j} \sum_{s} \lambda_{ij}^{rs} bc_{ii}^{r}}{\sum_{j} \sum_{s} \lambda_{ij}^{rs} (bc_{ij}^{r} - bc_{ii}^{r})} (\ln P_{i,t}^{r} C_{i,t}^{r} + \ln \pi_{ii,t}^{r}) - \ln P_{i,t}^{r} Y_{i,t}^{r}$$

In deviations from the steady state, HOT is given by

$$\ln \operatorname{HOT}_{i,t}^{r} = \frac{1 - \operatorname{HOT}_{i}^{r}}{\operatorname{HOT}_{i}^{r}} \left(\ln \operatorname{P}_{i,t}^{r} \operatorname{Y}_{i,t}^{r} - \ln(\operatorname{P}_{i,t}^{r} \operatorname{Y}_{i,t}^{r})_{\operatorname{DOM}} \right)$$

where $\ln P_{i,t}^r Y_{i,t}^r$ is the typical element of the vector $(\mathcal{P} + \mathbf{I}) \mathbf{\Lambda}^{-1} \ln \mathbf{Z}_t$, and $\ln(P_{i,t}^r Y_{i,t}^r)_{\text{DOM}}$ is computed using the block diagonal versions of the same matrices, focused on purely domestic linkages.

Appendix C

C.1 Simulations: Robustness checks

	(1)	(2)	(3)	(4)	(5)
$\ln \mathrm{HOT}$	0.016***				0.005***
	(0.001)				(0.001)
ln V		0 080***			0.006
$\Pi \Lambda$		0.089			0.000
		(0.005)			(0.004)
$\ln \phi$			-0.027***		-0.004
1			(0.009)		(0.016)
1 - (1/1)				0 177***	0 1/(+++
$\ln T(VA)$				-0.1//***	-0.166***
				(0.003)	(0.004)
0	1 68	1.68	1.68	1.68	1 68
ρ	1.00	1.00	1.00	1.00	1.00
ϵ	0.89	0.89	0.89	0.89	0.89
ψ	0.723	0.723	0.723	0.723	0.723
Obs.	2,002	2,000	2,018	2,018	1,987

Table C.1: Simulations results: : Robustness, $\rho = 1.68$, $\epsilon = 0.89$

Note: The dependent variable is simulated $\ln V_{i,t}^r$. All the regressors are defined in the text. Standard errors in parentheses. The coefficient of $\ln \phi$ is multiplied by 10,000 for legibility.

	(1)	(2)	(3)	(4)	(5)
$\ln \mathrm{HOT}$	0.024***				0.015***
	(0.001)				(0.001)
$\ln X$		0.115***			0.099***
		(0.003)			(0.003)
$\ln \phi$			-0.048***		-0.009
			(0.016)		(0.021)
$\ln T(VA)$				-0.247***	-0.205***
				(0.017)	(0.013)
ρ	1.43	1.43	1.43	1.43	1.43
ϵ	1.20	1.20	1.20	1.20	1.20
ψ	0.723	0.723	0.723	0.723	0.723
Obs.	2,002	2,000	2,018	2,018	1,987

Table C.2: Simulations results: Robustness, $\rho = 1.43$, $\epsilon = 1.20$

Note: The dependent variable is simulated $\ln V_{i,t}^r$. All the regressors are defined in the text. Standard errors in parentheses. The coefficient of $\ln \phi$ is multiplied by 10,000 for legibility.

	(1)	(2)	(3)	(4)	(5)
ln HOT	0.021***				0.009***
	(0.001)				(0.001)
ln X		0.094***			0.061***
		(0.003)			(0.002)
$\ln \phi$			-0.037**		-0.005
T			(0.015)		(0.014)
ln T(VA)				-0.326***	-0.216***
				(0.009)	(0.008)
0	1.68	1 68	1 68	1.68	1 68
ϵ	1.20	1.20	1.20	1.20	1.20
ψ	0.723	0.723	0.723	0.723	0.723
Obs.	2,002	2,000	2,018	2,018	1,987

Table C.3: Simulations results: Robustness, $\rho = 1.68$, $\epsilon = 1.20$

Note: The dependent variable is simulated $\ln V_{i,t}^r$. All the regressors are defined in the text. Standard errors in parentheses. The coefficient of $\ln \phi$ is multiplied by 10,000 for legibility.

Appendix D

D.1 HOT

The WIOD dataset spans the years 2000 - 2014. The data covers 44 countries (including a "rest of the world") and 56 sectors classified according to the International Standard Industrial Classification (ISIC) revision 4. The data are available at wiod.org. The method to calculate HOT is described in Section **??** and the method to calculate the instrument for HOT can be found in Section **3.2**.

D.2 Value Added

Value added is converted in PPP USD and deflated using industry price levels of gross value added. Value added is in millions of national currency, price levels are indexed at 2010 = 100. All data are sourced from WIOD Socio-Economic Accounts (SEA). PPP USD exchange rates are sourced from the IMF.

D.3 Growth

Growth is constructed as the logarithm of sector level value added growth per employee, expressed in real PPP USD. Value added is in national currency and converted in USD at PPP exchange rate; it is deflated using industry price indices of gross value added. The data are sourced from WIOD SEA and the IMF.

D.4 Business Cycles Synchronization

SYNC1 is the demeaned product of real value added growth between country-sector pairs divided by each country-sector standard deviations. SYNC2 is measured as minus the absolute pairwise difference in the logarithm of real value added growth between country-sector pairs, measured each year. Value added is in national currency and converted in USD at PPP exchange rate. It is deflated using industry price indices. The source of the data are the WIOD SEA and the IMF.

D.5 Direct Trade measures: X and ϕ

Direct exports, X, are given by the ratio of total exports of intermediate and final goods to value added for each country-sector. Both numerator and denominator are expressed in current USD at PPP exchange rates. The bilateral version of X is given by the ratio of $PM_{ij}^{rs} + PM_{ji}^{rs}$ to $VA_i^r + VA_j^r$ for lack of data on bilateral trade in final goods. Both numerator and denominator are expressed in current PPP USD. ϕ is defined in section 2.3, and all its components are measured in PPP USD. Intermediate goods exports and final goods exports are obtained from WIOD's World Input-Output Tables. Value added is in national currency and converted in USD at PPP exchange rate. Value added is sourced from WIOD SEA and PPP exchange rate from the IMF.

D.6 Trade in Value Added (TiVA): T_i^r

The variants of TiVA used in the paper, $T_i^r(X)$ and $T_i^r(VA)$ are described in section 2.3. TiVA measures are constructed using the Input-Output Tables from WIOD. $T_i^r(VA)$ is normalized by Value Added in real PPP USD. Value added is sourced from WIOD SEA and PPP exchange rate from the IMF. $T_i^r(X)$ is normalized by gross exports which are the sum of intermediate and final exports found in World Input-Output Tables provided by WIOD.