

1 Supplementary materials: details of proofs*

2 Jean Imbs[†] Eric Jondeau[‡] Florian Pelgrin[§]

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[†](Corresponding author) Paris School of Economics and CEPR. Postal address: Paris School of Economics, 106 Boulevard de l'Hopital, 75013 Paris, France. Email: Jean.Imbs@parisschoolofeconomics.eu

[‡]Faculty of Business and Economics, University of Lausanne, and Swiss Finance Institute. Postal address: University of Lausanne, CH-1015 Lausanne, Switzerland. Email: Eric.Jondeau@unil.ch

[§]Faculty of Business and Economics, University of Lausanne and CIRANO. Postal address: University of Lausanne, CH-1015 Lausanne, Switzerland. Email: Florian.Pelgrin@unil.ch

4 **Appendix 1: Derivation of the model with marginal costs following an AR(2)**

5 The full model of (sectoral) inflation rests on the following system

$$\begin{aligned}
 6 \quad \hat{\pi}_{j,t} &= \lambda_j^b \hat{\pi}_{j,t-1} + \lambda_j^f E_t \hat{\pi}_{j,t+1} + \theta_j \hat{s}_{j,t}^{avg} + \varepsilon_{j,t}^\pi \\
 7 \quad \hat{s}_{j,t}^{avg} &= \rho_{1j} \hat{s}_{j,t-1}^{avg} + \rho_{2j} \hat{s}_{j,t-2}^{avg} + u_{j,t}.
 \end{aligned}$$

8 Using the companion form of the second equation, we have

$$\begin{aligned}
 9 \quad \pi_{j,t} &= \lambda_j^b \hat{\pi}_{j,t-1} + \lambda_j^f E_t \hat{\pi}_{j,t+1} + \begin{pmatrix} \theta_j & 0 \end{pmatrix} \begin{pmatrix} \hat{s}_{j,t}^{avg} \\ \hat{s}_{j,t-1}^{avg} \end{pmatrix} + \varepsilon_{j,t}^\pi \\
 10 \quad \begin{pmatrix} \hat{s}_{j,t}^{avg} \\ \hat{s}_{j,t-1}^{avg} \end{pmatrix} &= \Lambda_j \begin{pmatrix} \hat{s}_{j,t}^{avg} \\ \hat{s}_{j,t-1}^{avg} \end{pmatrix} + \begin{pmatrix} u_{j,t} \\ 0 \end{pmatrix}
 \end{aligned}$$

11 where

$$12 \quad \Lambda_j = \begin{pmatrix} \rho_{1j} & \rho_{2j} \\ 1 & 0 \end{pmatrix}.$$

13 Determining the roots of the characteristic equation, we have

$$14 \quad \hat{\pi}_{j,t} = \delta_{1j} \hat{\pi}_{j,t-1} + \frac{1}{\delta_{2j} \lambda_j^f} \begin{pmatrix} \theta_j & 0 \end{pmatrix} (I_2 - \delta_{2j}^{-1} \Lambda_j)^{-1} \begin{pmatrix} \hat{s}_{j,t}^{avg} \\ \hat{s}_{j,t-1}^{avg} \end{pmatrix} + \tilde{\varepsilon}_{j,t}^\pi$$

15 where

$$16 \quad \delta_{1j} = \frac{1 - \sqrt{1 - 4\lambda_j^f \lambda_j^b}}{2\lambda_j^f} \quad \text{and} \quad \delta_{2j} = \frac{1 + \sqrt{1 - 4\lambda_j^f \lambda_j^b}}{2\lambda_j^f}.$$

17 It is straightforward to show that the sectoral reduced-form can be expressed as

$$18 \quad \hat{\pi}_{j,t} = \delta_{1j} \hat{\pi}_{j,t-1} + \phi_{1j} \hat{s}_{j,t}^{avg} + \phi_{2j} \hat{s}_{j,t-1}^{avg} + \tilde{\varepsilon}_{j,t}^\pi$$

19 where

$$20 \quad \phi_{1j} = \frac{\theta_j}{\Delta_j \delta_{2j} \lambda_j^f}$$

$$21 \quad \phi_{2j} = \phi_{1j} \frac{\rho_{2j}}{\delta_{2j}}$$

$$22 \quad \Delta_j = 1 - \frac{\rho_{1j}}{\delta_{2j}} - \frac{\rho_{2j}}{\delta_{2j}^2}.$$

23 Appendix 2: Sectoral Phillips Curves with Common Correlated Effects

24 With common correlated effects, the model in equation (9) can be rewritten:

$$25 \quad \hat{\pi}_{j,t} = \delta_{1j} \hat{\pi}_{j,t-1} + \phi_{1j} h_j \hat{s}_{j,t}^{avg} + \phi_{2j} h_j \hat{s}_{j,t-1}^{avg} + f_t' \gamma_{\pi,j} + \bar{\varepsilon}_{j,t}^{\pi} \quad (1)$$

$$26 \quad \hat{s}_{j,t}^{avg} = \rho_{1j} \hat{s}_{j,t-1}^{avg} + \rho_{2j} \hat{s}_{j,t-2}^{avg} + f_t' \gamma_{s,j} + \bar{u}_{j,t},$$

27 where $f_t = (\bar{\pi}_t, \bar{\pi}_{t-1}, \bar{s}_t, \bar{s}_{t-1}, \bar{s}_{t-2})'$, \bar{x}_t is the cross-sectional average of $\hat{x}_{j,t}$, and $\bar{\varepsilon}_{j,t}^{\pi}$ (re-
28 spectively $\bar{u}_{j,t}$) denotes an independent and identically distributed shock to inflation (re-
29 spectively real marginal costs) in sector j .

30 Appendix 3: Robustness - Sectoral Estimates

- 31 • Table A1: Sectoral Phillips Curves with GMM;
- 32 • Table A2: Sectoral Phillips Curves with Common Correlated Effect.

Table A1: Sectoral Phillips Curves with GMM

Industries	Agri.	Food	Cons.	Car	Equip.	Interm.	Energy	Const.	Trade	Transp.	Fin.	Real Estate	Business	Pers.	Serv.	Educ. & Health	Govt
Reduced Form - Equation (??)																	
λ^b	0.498*** (0.028)	0.000 (0.385)	0.153* (0.085)	0.291*** (0.079)	0.302*** (0.090)	0.338*** (0.052)	0.000 (0.057)	0.414*** (0.024)	0.387*** (0.052)	0.000 (0.117)	0.484*** (0.030)	0.493*** (0.064)	0.000 (0.168)	0.120* (0.070)	0.482*** (0.012)	0.501*** (0.019)	0.484*** (0.009)
λ^f	0.501*** (0.047)	0.999*** (0.385)	0.632*** (0.094)	0.540*** (0.197)	0.697*** (0.131)	0.648*** (0.130)	0.693*** (0.104)	0.515*** (0.042)	0.572*** (0.124)	0.974*** (0.262)	0.515*** (0.054)	0.507*** (0.081)	0.999*** (0.374)	0.879*** (0.118)	0.501*** (0.019)	0.516*** (0.012)	0.516*** (0.012)
θ	0.934 (0.905)	2.551* (1.426)	0.801** (0.346)	0.020 (0.037)	0.189 (0.214)	0.033 (0.177)	7.460*** (2.299)	0.134 (0.115)	0.143 (0.308)	0.229 (1.486)	0.207 (0.219)	0.023 (1.148)	0.195 (1.295)	0.253 (0.384)	0.028 (0.045)	0.022* (0.0012)	0.022* (0.0012)
Structural Estimates																	
ω	0.237*** (0.114)	0.000 (0.047)	0.120** (0.048)	0.463*** (0.108)	0.238** (0.105)	0.393* (0.214)	0.016* (0.009)	0.427*** (0.087)	0.371*** (0.123)	0.005 (0.103)	0.417*** (0.121)	0.671 (0.459)	0.000 (0.088)	0.762 (0.878)	0.647*** (0.133)	0.647*** (0.133)	0.659*** (0.046)
α	0.239*** (0.124)	0.232* (0.123)	0.349*** (0.072)	0.782*** (0.230)	0.558*** (0.162)	0.754* (0.451)	0.093*** (0.022)	0.529*** (0.123)	0.545* (0.327)	0.623 (0.964)	0.448*** (0.151)	0.698 (0.457)	0.649 (0.906)	0.579*** (0.215)	0.678*** (0.161)	0.678*** (0.161)	0.709*** (0.055)
Duration	1.314*** (0.215)	1.304*** (0.210)	1.535*** (0.174)	4.589 (4.848)	2.262** (0.828)	4.059*** (0.674)	1.102*** (0.026)	2.125*** (0.554)	2.201 (1.636)	2.650 (6.776)	1.812*** (0.475)	3.310 (5.103)	2.851 (7.368)	2.378*** (1.219)	3.108* (1.557)	3.439*** (0.654)	3.439*** (0.654)

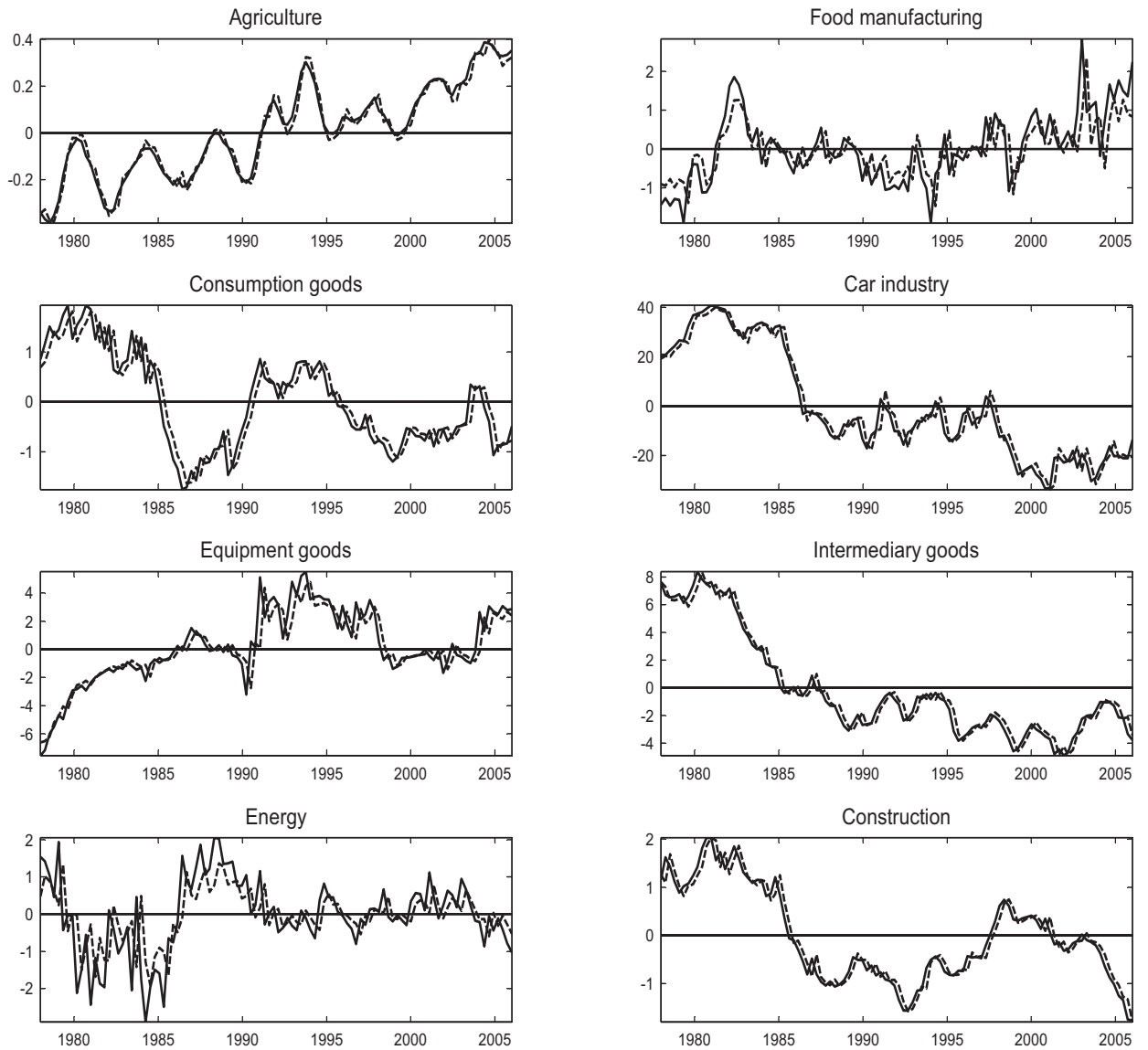
Note: Standard deviation in parentheses. The estimates are obtained using the generalized method of moments.

Table A2: Sectoral Phillips Curves with Common Correlated Effect

Industries	Agr.	Food	Cons.	Car	Equip.	Interm.	Energy	Const.	Trade	Transp.	Fin.	R. Estate	Business Pers. Serv.	Educ. & Health	Govt	
	Reduced Form - Equation (2)															
δ_1	0.776*** (0.062)	0.002 (0.084)	0.453*** (0.091)	0.213*** (0.086)	0.054 (0.083)	0.361*** (0.071)	0.003 (0.046)	0.370*** (0.081)	0.468*** (0.076)	0.001 (0.041)	0.485*** (0.073)	0.580*** (0.076)	0.001 (0.078)	0.313*** (0.081)	0.665*** (0.074)	0.795*** (0.059)
ϕ_1	27.131** (11.138)	1.716* (1.032)	0.357 (0.722)	0.569* (0.290)	0.348 (0.311)	0.908** (0.362)	4.541*** (1.703)	0.908 (0.837)	2.042*** (0.811)	0.001 (0.077)	2.645** (1.246)	7.645 (15.57)	0.001 (0.009)	0.001 (0.013)	0.765** (0.307)	0.004 (0.059)
ϕ_2	-17.68** (7.327)	0.057 (0.138)	0.015 (0.041)	0.029 (0.048)	0.032 (0.037)	-0.097 (0.082)	1.004** (0.404)	-0.191 (0.192)	-0.484* (0.264)	-0.001 (0.002)	-1.465** (0.704)	-5.473 (11.17)	0.001 (0.003)	-0.001 (0.003)	0.146* (0.079)	0.001 (0.013)
ρ_1	1.586*** (0.059)	0.852*** (0.097)	0.866*** (0.092)	0.832*** (0.088)	0.846*** (0.086)	1.053*** (0.079)	0.427*** (0.008)	1.157*** (0.081)	1.123*** (0.082)	0.609*** (0.078)	1.504*** (0.062)	1.648*** (0.069)	0.520*** (0.069)	1.118*** (0.092)	0.729*** (0.089)	0.727*** (0.096)
ρ_2	-0.675*** (0.053)	0.034 (0.098)	0.044 (0.094)	0.051 (0.085)	0.093 (0.081)	-0.108 (0.076)	0.223 (0.057)	-0.213*** (0.079)	-0.240*** (0.076)	-0.026 (0.040)	-0.561*** (0.056)	-0.729*** (0.068)	0.275*** (0.066)	-0.184** (0.095)	0.193* (0.100)	0.216*** (0.099)
	Reduced Form - Equation (??)															
λ^b	0.441*** (0.022)	0.002 (0.083)	0.313*** (0.043)	0.175*** (0.059)	0.051 (0.075)	0.267*** (0.039)	0.003 (0.046)	0.271*** (0.044)	0.320*** (0.036)	0.001 (0.041)	0.328*** (0.034)	0.369*** (0.031)	0.001 (0.077)	0.239*** (0.047)	0.402*** (0.027)	0.445*** (0.019)
λ^f	0.554*** (0.021)	0.989*** (0.082)	0.682*** (0.043)	0.817*** (0.058)	0.940*** (0.074)	0.728*** (0.038)	0.987*** (0.045)	0.724*** (0.043)	0.675*** (0.035)	0.989*** (0.039)	0.667*** (0.033)	0.626*** (0.031)	0.989*** (0.076)	0.756*** (0.046)	0.595*** (0.027)	0.553*** (0.018)
θ	1.533* (0.801)	0.211 (0.222)	0.025 (0.058)	0.060 (0.046)	0.024 (0.028)	0.043 (0.034)	1.619* (0.952)	0.043 (0.052)	0.177* (0.101)	0.001 (0.003)	0.112 (0.084)	0.413 (0.871)	0.000 (0.002)	0.001 (0.001)	0.042* (0.022)	0.001 (0.002)
	Structural Estimates															
ω	0.156*** (0.047)	0.001 (0.053)	0.359*** (0.102)	0.159* (0.064)	0.046 (0.071)	0.273*** (0.055)	0.001 (0.015)	0.280*** (0.068)	0.262*** (0.052)	0.001 (0.040)	0.302*** (0.052)	0.239 (0.188)	0.001 (0.075)	0.313*** (0.089)	0.465*** (0.067)	0.772*** (0.019)
α	0.198*** (0.064)	0.636*** (0.146)	0.793*** (0.207)	0.749*** (0.084)	0.855*** (0.084)	0.756*** (0.082)	0.301*** (0.075)	0.756*** (0.127)	0.559*** (0.097)	0.999*** (0.272)	0.620*** (0.106)	0.409 (0.322)	0.999*** (0.174)	0.999*** (0.122)	0.694*** (0.071)	0.970*** (0.236)
Duration	1.247*** (0.099)	2.750** (1.105)	4.819*** (0.316)	3.991** (1.332)	6.888* (3.994)	4.098*** (1.369)	1.432*** (0.038)	4.104* (2.137)	2.267*** (0.496)	-	2.628*** (0.729)	1.692* (0.923)	-	-	3.272*** (0.766)	33.286 (262.52)

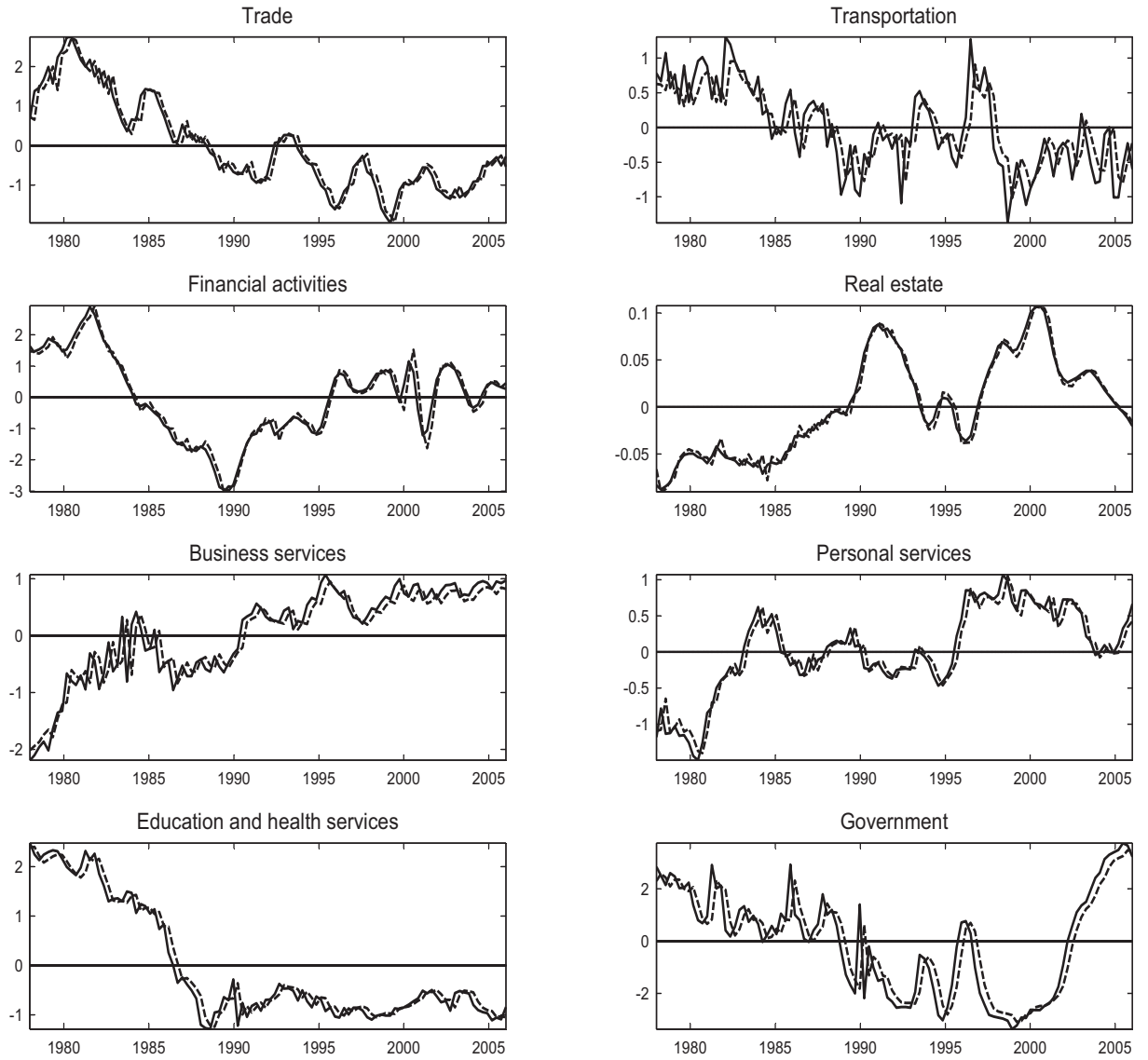
Note: Standard deviation in parentheses. Marginal costs follow an autoregressive process of order two. The estimates are obtained using a Maximum Likelihood procedure, including Common Correlated Effects. The structural estimates are inferred from reduced form coefficients using a Delta method.

Figure 1: Real Marginal Cost and AR(2)



Note: The plain line denotes observed marginal costs and the dotted line is the fitted AR(2).

Figure 1 (continued): Real Marginal Cost and AR(2)



Note: The plain line denotes observed marginal costs and the dotted line is the fitted AR(2).